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Dynamical evolution of Saturn's F ring dust particles

R. Sfair,★ S. M. Giuliatti Winter, D. C. Mourão and O. C. Winter

UNESP – São Paulo State University, Guaratinguetá, CEP 12516-410, SP, Brazil

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ABSTRACT

Saturn's F ring has been the subject of study due to its peculiar structure and the proximity to two satellites, named Prometheus (interior) and Pandora (exterior to the ring), which cause perturbations to the ring particles. Early results from Voyager data have proposed that the ring is populated with centimetre- and micrometre-sized particles. The Cassini spacecraft also detected a less dense part in the ring with width of 700 km. Small particles suffer the effects of solar radiation. Burns et al. showed that due to effects of one component of the solar radiation, the Poynting–Robertson drag, a ring particle will decay in the direction of the planet in a time much shorter than the age of the Solar system. In this work, we have analysed a sample of dust particles (1, 3, 5 and 10 μm) under the effects of solar radiation, the Poynting–Robertson drag and the radiation pressure components and the gravitational effects of the satellites Prometheus and Pandora. In this case, the high increase of the eccentricity of the particles leads almost all of them to collide with the outer edge of the A ring. The inclusion of the oblateness of Saturn in this system significantly changes the outcome, since the large variation of the eccentricity is reduced by the oblateness effect. As a result, there is an increase in the lifetime of the particle in the envelope region. Our results show that even the small dust particles, which are very sensitive to the effects of solar radiation, have an orbital evolution similar to larger particles located in the F ring. The fate of all particles is a collision with Prometheus or Pandora in less than 30 years. On the other hand, collisions of these particles with moonlets/clumps present in the F ring could change this scenario.

Key words: Solar system: general.

1 INTRODUCTION

The region which encompasses the orbits of the F ring and the close satellites Prometheus and Pandora has been studied since the Voyager spacecraft encountered the Saturnian System in 1980 and 1981. The unique structure of the F ring can be seen in details through the data sent by the Cassini spacecraft (Porco et al. 2005).

Modelling the F ring population has also been a challenge. Showalter et al. (1992) have proposed a model for the F ring population by using photometric observations and occultation data. Their model consists of a ring composed by centimetre-sized particles in a small core of about 1-km-wide and micrometre-sized dust in a much wider 'envelope' (about 500 km wide). They proposed that this core has an equivalent mass to a moonlet of 15–70 km radius distributed along the ring in a large number of small bodies.

This population of dust particles located in the envelope region of the F ring suffers the effects of perturbative forces. As has been analysed by Burns, Lamy & Soter (1979), the effects of the solar radiation can be divided into two components: the radiation pressure (RP) and the Poynting Robertson (PR) drag. If the two components of the solar radiation could be separate, we could expect two different responses in the orbital motion of a dust particle. Under the effects of the RP, the eccentricity of the particle will oscillate and under the effects of the PR drag, which removes the orbital angular momentum from the particle through the absorption and the re-emission of the solar radiation, the semimajor axis of the particle will decrease until collision with the planet in a time less than the age of the Solar system (Burns et al. 1984).

The Cassini spacecraft sent several F ring images showing a 700-km-wide envelope which could be seen through the enhanced images (Porco et al. 2005). However, no work has been published so far to provide any further information regarding the size of these particles located in the envelope region.

Murray et al. (2005) have included in their numerical simulations of the F ring an envelope composed of dust particles. They showed that at some configurations Prometheus can enter the envelope region.

The goal of this paper is to analyse the orbital evolution of the dust particles located at the Saturnian F ring taking into account the solar radiation, the gravitational perturbations of Prometheus and Pandora and the oblateness of Saturn. In the next section, we

★E-mail: rsfair@feg.unesp.br

derive the components of the solar radiation force; and in Section 3 we present our results. The conclusions are summarized in the last section.

2 SOLAR RADIATION FORCE

The dynamical system considered here is within the framework of the planar four-body problem, planet–satellites–ring particle and a time-dependent force. We have also included the planet's non-spherical mass distribution taking into account the quadruple term J_2, all other terms have been neglected. The vectorial expression for the solar radiation force on a particle around a planet is given by (Mignard 1984)

$$\boldsymbol{F} = \beta \left\{ \frac{\boldsymbol{r}_{\rm sp}}{r_{\rm sp}} \left[1 - \frac{\boldsymbol{r}_{\rm sp}}{r_{\rm sp}} \left(\frac{\boldsymbol{v}_{\rm p}}{c} + \frac{\boldsymbol{v}}{c} \right) \right] - \left(\frac{\boldsymbol{v}_{\rm p}}{c} + \frac{\boldsymbol{v}}{c} \right) \right\}, \quad (1)$$

where c is the speed of light, $\boldsymbol{v}$ is the velocity vector of the particle relative to the planet, $\boldsymbol{r}_{\rm sp}$ is the Sun–planet position vector, $\boldsymbol{v}_{\rm p}$ is the velocity vector of the planet relative to the Sun and $r_{\rm sp} = |\boldsymbol{r}_{\rm sp}|$. The first term corresponds to the RP component and the other terms to the PR drag component. We assumed spherical particles which obey geometrical optics. The value of β is given by (Burns et al. 1979)

$$\beta = 5.7 \times 10^{-5} \frac{Q_{\rm PR}}{\rho s}, \quad (2)$$

where s and ρ are the radius and the density of a spherical particle in cgs units and $Q_{\rm PR}$ is a constant value which depends on the optical properties of the grain.

The components of the solar radiation force can be given by

$$F_x = \frac{\beta G M_{\rm s}}{r_{\rm sp}^2} \left[\cos(n_{\rm s} t) - \left(\frac{x_{\rm s}}{r_{\rm sp}} \right)^2 \left(\frac{v_{x{\rm s}}}{c} + \frac{v_x}{c} \right) - \left(\frac{v_{x{\rm s}}}{c} + \frac{v_x}{c} \right) \right] \quad (3)$$

$$F_y = \frac{\beta G M_{\rm s}}{r_{\rm sp}^2} \left[\sin(n_{\rm s} t) - \left(\frac{y_{\rm s}}{r_{\rm sp}} \right)^2 \left(\frac{v_{y{\rm s}}}{c} + \frac{v_y}{c} \right) - \left(\frac{v_{y{\rm s}}}{c} + \frac{v_y}{c} \right) \right], \quad (4)$$

where G is the gravitational constant, $M_{\rm s}$ and $n_{\rm s}$ are the mass and the mean motion of the Sun, respectively, $(x_{\rm s}, y_{\rm s})$ are the coordinates of the Sun–planet position, (v_x, v_y) are the components of the velocity of the particle. The subscript 's' in the velocity components refers to the Sun. The first term corresponds to the RP component and the other terms to the PR drag component. We have neglected the planetary shadow and the light reflected from the planet, since both effects are at least one order of magnitude weaker than the effect caused by the solar radiation (Hamilton & Krivov 1996).

3 NUMERICAL SIMULATIONS

We have numerically simulated a sample of dust particles initially located in the envelope region of the F ring. This region is 700 km wide and has a mean semimajor axis and eccentricity equal to the core of the F ring (Murray et al. 2005). These dust particles are disturbed by the gravitational perturbations of Prometheus and Pandora, the oblateness of Saturn (J_2 term (Jacobson et al. 2006)) and the RP and PR drag components.

The initial positions of the satellites adopted in the numerical simulations were derived from Jacobson et al. (2008). The values of the mass (m), semimajor axis (a), eccentricity (e), longitude of pericentre (ϖ) and mean anomaly (M) of Prometheus and Pandora are listed in Table 1.

Table 1. The parameters of the satellites Prometheus and Pandora (Jacobson et al. 2008), and the F ring core (Murray et al. 2005).

Parameters	Prometheus	Pandora	F ring core
m (kg)	1.59×10^{17}	1.37×10^{17}	–
a (km)	139 380	141 710	140 224
$e(\times 10^{-3})$	2.2	4.2	2.6
$\varpi(°)$	161.0	83.4	–
M (°)	242.3	202.5	–

The sizes of the particles were chosen to be 1, 3, 5 and 10 μm. For each particle size, we have simulated 100 particles with initial semimajor axis uniformly distributed in the envelope region of the F ring (700 km). The other initial orbital parameters of the particles were $e = 2.6 \times 10^{-3}$, $M = 0$ and $\varpi = 0$.

The numerical simulations were performed using the Bulirsh–Stöer numerical integrator from the Mercury package (Chambers 1999). Some modifications were necessary in order to include the solar radiation force.

3.1 Without the effects of Saturn's oblateness

The effect of each component of the solar radiation was analysed separately. Fig. 1 shows the time evolution of the semimajor axis and eccentricity of four particles with size of 1, 3, 5 and 10 μm with the same initial conditions. The semimajor axis of the F ring's core is located at 0. As expected, the PR drag component is responsible for the decrease in semimajor axis of each particle leading to a collision with the planet. The rate of decrease of the semimajor axis depends on the size of the particle, the 1-μm-sized particle decays 100 km in about 60 years. An extrapolation of the decay time obtained in Fig. 1a shows that the particles collide with Saturn in 5×10^4 and 4×10^5 yr, considering 1- and 10-μm-sized particles, respectively. These values have the same magnitude of the estimation derived by Burns et al. (1979) (cf. their equation 55).

The RP component provokes an oscillation in the semimajor axis and a large variation in the eccentricities (Fig. 1b) of the particles. For 1-μm-sized particle, the eccentricity reaches large values (about 0.6), leading to a collision with the planet in less than 10 years. For larger particles, the eccentricity oscillates with a period equal to the orbital period of Saturn.

When both components of the solar radiation and the gravitational influence of the satellites Prometheus and Pandora are taken into account, the behaviour of the semimajor axis of the particles changes. Fig. 2 shows the orbital evolution of two particles, of 1 and 10 μm in size, initially located at the semimajor axis of the core of the F ring. By comparing Figs 1(a) and 2(a), we verified that the semimajor axis of both particles (fig. 2a) oscillates due to RP component. The smaller particle has an amplitude of oscillation of about 10 km. The temporal evolution of the semimajor axis of both particles also shows 'jumps' (Fig. 2a) which are associated with close approaches with one of the satellites (Winter et al. 2007). However, the temporal variations of the eccentricities presented in Figs 1(b) and 2(b) are similar, since, according to other numerical simulation that we have performed, the gravitational effects of the satellites on the eccentricities of the particles are small [$\mathcal{O}(10^{-5})$] when compared with the effects of the RP component. The vertical

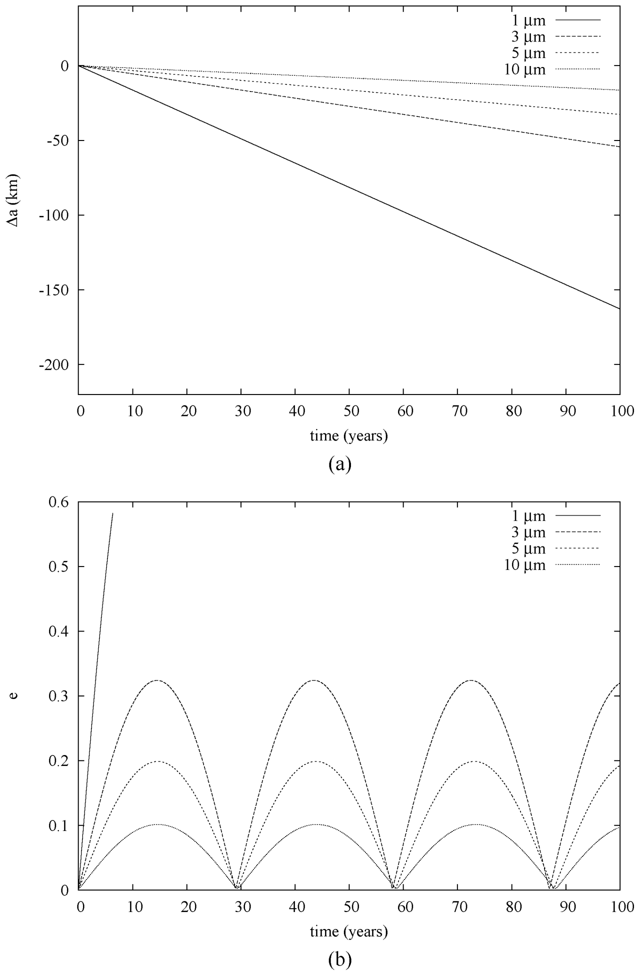


Figure 1. Time evolution of the (a) semimajor axis due to the effects of the PR drag only and (b) the eccentricity due to the RP component of four particles of different sizes. The semimajor axis of the F ring's core is located at 0.

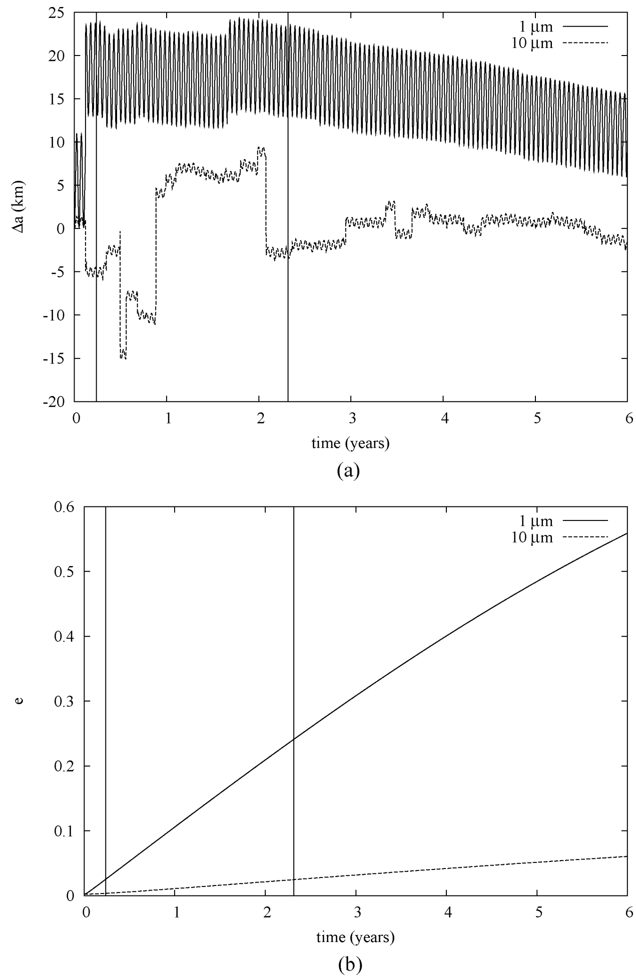


Figure 2. Evolution of the (a) semimajor axis and (b) eccentricity of 1- and 10-μm-sized particles under the effect of the solar radiation and the gravitational perturbation of Prometheus and Pandora. The semimajor axis of the F ring's core is located at 0. Vertical lines represent the collision time between the particles and the A ring.

lines in Fig. 2 show the collision time between the particles and the A ring. The smaller particle collides with the A ring in less than ∼3 months and the larger particle in less than 2.3 years. We will discuss the collision time in the next section.

3.2 With the effects of Saturn's oblateness

In this section, we compare the effects caused by the inclusion of the gravity coefficient J_2 in the orbital evolution of a dust F ring particle. Fig. 3 shows the variation of the eccentricities of two particles, sizes of 1 and 10 μm, with identical initial conditions. As can be seen in Fig. 1(b), the RP component induces larger variation in the eccentricity of the smallest particle. For example, the eccentricity of the 1-μm-sized particle reaches the value 0.6, and for a 10-μm-sized particle the eccentricity increases up to 0.1. When the effect of Saturn's oblateness is considered, the amplitude of the eccentricity decreases to a value of ∼0.01 for both particles, as shown in Fig. 3. This result was explained by Hamilton & Krivov (1996). They analysed the effects on a dust particle when the planetary oblateness and the RP component act simultaneously. For the 1-μm-sized particle located up to 9 R_p (see their fig. 1), where R_p is the Saturnian radius, the effect of the planetary oblateness is stronger than the perturbation caused by the PR drag component.

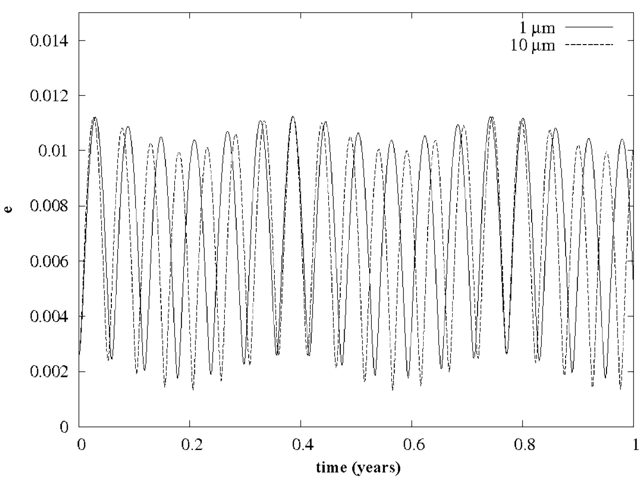
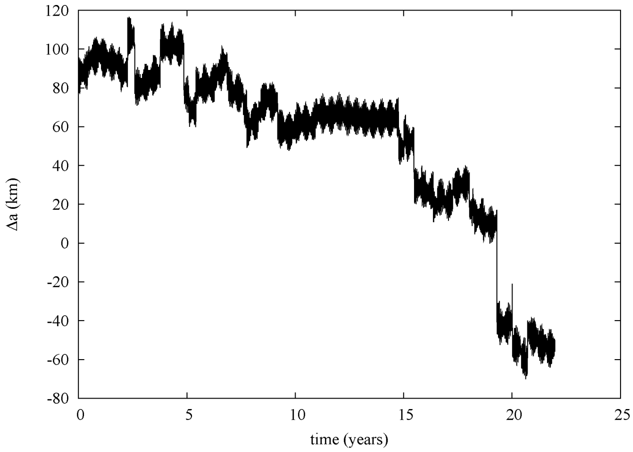
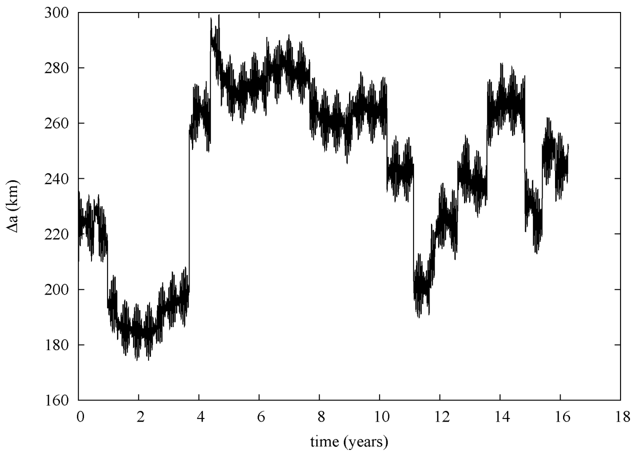


Figure 3. Variation of the eccentricity of two particles of size 1 and 10 μm disturbed only by the solar radiation and the oblateness of Saturn. The initial conditions of the two particles are identical.



(a)



(b)

Figure 4. Variation of the semimajor axis of (a) a 5-μm-sized particle and (b) a 3-μm-sized particle under the effects of the solar radiation, the gravitational perturbation of the satellites and the oblateness of Saturn. The particle represented in (a) collides with Prometheus, while the particle in (b) has a collision with Pandora. The semimajor axis of the F ring's core is located at 0.

The strength of the planetary oblateness decreases with the increase of the distance from the planet. On the other hand, the strength of the PR drag component is weaker for those particles located near the planet. The period of the variation of the eccentricities (less than 0.1 year) is much smaller than the period showed in our Fig. 1(b) (about 30 years).

In Fig. 4, the evolution of the semimajor axis of two representative examples of our numerical simulations for 5- and 3-μm-sized particles under the effects of the solar radiation force, the gravitational perturbation of Prometheus and Pandora and the oblateness of Saturn are presented. As can be seen in Fig. 1(a), a particle of 5 μm size decays ∼30 km in 100 years due to the PR drag component. The sudden changes in the semimajor axis of the particles, seen in Fig. 4, are due to close encounters between the particle and one of the satellites, which induces an increase or decrease of its value (Winter et al. 2007). Due to the oblateness of Saturn, the collision with one of the satellites will also depend on the variation of the semimajor axis, since larger variations of the eccentricity, due to the RP component, are damped by the effects caused by the oblateness.

We have computed the number of the particles that collide with one of the satellites or with the outer edge of the A ring, located at ∼136 780 km from Saturn. We have also calculated the mean collision time, $\langle T_c \rangle$, and its standard deviation. Table 2 presents these results for those particles under the gravitational effects of the satellites and the solar radiation. As can be seen, most of the particles collide with the outer edge of the A ring in less than 3 years. Table 3 presents the results for the numerical simulations of the system that also took into account the oblateness of Saturn. A comparison of Tables 2 and 3 shows a remarkable difference on the fate of the particles. The effects of the oblateness of the planet are such that the particles collide with one of the satellites before reaching the outer edge of the A ring. This is due to the fact that the oblateness damps the variation of the eccentricity. A ring particle located at the outer edge of the envelope needs an eccentricity of ∼0.025 to cross the outer edge of the A ring. However, the variation of the eccentricity, when the oblateness is taking into account, reaches a maximum value ∼0.012 (Fig. 3), avoiding the collision with the outer edge of the A ring. By comparing Tables 2 and 3, it can be seen that there is an increase of the collision time, when the oblateness of the planet is included, independently of the size of the particle.

Table 2. The fate of the dust particles under the gravitational effects of the satellites and the solar radiation.

Particle (μm)	Prometheus collision	Pandora collision	A ring collision	$\langle T_c \rangle$ (yr)
1	3	1	96	0.25(±0.03)
3	2	6	92	0.7(±0.1)
5	9	10	81	1.1(±0.25)
10	20	15	65	1.9(±0.6)

Table 3. The fate of the dust particles when the oblateness of Saturn is also taken into account.

Particle (μm)	Prometheus collision	Pandora collision	A ring collision	$\langle T_c \rangle$ (yr)
1	57	43	0	6.5(±5.8)
3	74	26	0	5.2(±4.8)
5	76	24	0	7.3(±6.2)
10	68	32	0	7.7(±6.5)

From Table 3 we have also noted that about 70 per cent of the particles collide with Prometheus, since this satellite is closer to the ring. The only exception occurs for 1-μm-sized particle, because its semimajor axis presents a larger oscillation than the semimajor axis of the other particles, which is caused by the PR drag component.

Fig. 5 shows the collisional time (T_c) as a function of Δa_0 for a sample of particles of 1, 3, 5 and 10 μm in size. The F ring's core is located at 0. The full lines, in Fig. 5, represent those particles under the gravitational effects of Prometheus and Pandora and the solar radiation pressure. Almost all particles collide with the outer edge of the A ring in a time less than 3 years. The increase of the eccentricities, due to the RP component, of these tiny particles leads to a large variation in the orbital radius which allows the orbits of the particles to cross the outer edge of the A ring. In Fig. 5, those particles represented by the dashed line are also perturbed by the oblateness. When this effect is included, the collision time increases for all particles.

The difference in the collision time is due to the fact that the planetary oblateness damps the eccentricity caused by the RP component. Our results show that when the oblateness of Saturn is present almost 70 per cent of the particles collide with Prometheus. For this case, the variation of the orbital radius is smaller

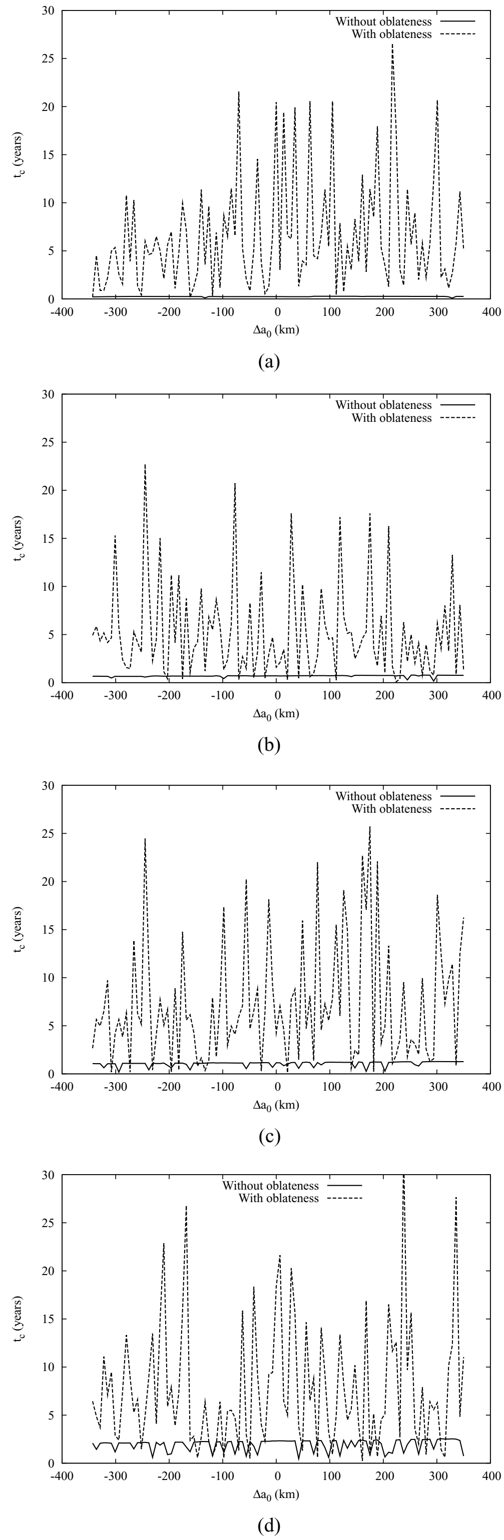


Figure 5. Collision time (t_c) for particles of different sizes: (a) 1-μm-, (b) 3-μm-, (c) 5-μm- and (d) 10-μm-sized particles as a function of the initial semimajor axis a_0. Dashed lines represent those particles perturbed by the solar radiation pressure and Prometheus and Pandora, and the full lines represent those particles perturbed also by the J_2 term. The semimajor axis of the F ring's core is located at 0.

allowing more collisions with Prometheus since it is closer to the envelope region.

4 FINAL COMMENTS

In this work, we have analysed the behaviour of a sample of F ring dust particles under the effects of the solar radiation force and the gravitational perturbations of Prometheus and Pandora, in orbit around an oblate Saturn.

When the oblateness of Saturn is not considered, the large variation of the eccentricity, induced by the RP component, leads most of the particles to a collision with the outer edge of the A ring. However, the variation in the eccentricity of the F ring particles is decreased by the effect of the planetary oblateness. As a result, there is an increase in the lifetime of the particle in the envelope region.

Our results show that the dust particles, which are very sensitive to the effects of the solar radiation pressure, have an orbital evolution similar to the population of large particles.

Most of these particles (about 70 per cent) collide with Prometheus in less than 30 years. It is important to point out that these dust particles spend most of the time, before colliding with the satellites, wandering in the F ring region. Cassini data showed that moonlets and clumps populate the F ring (Porco et al. 2005). The paper by Winter et al. (2007) has shown that these bodies are in confined chaotic orbits. Therefore, the dust particles can collide with these moonlets/clumps before leaving the F ring region.

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